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Modeling and optimizing bus transit priority along an arterial: A moving bottleneck approach

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ABSTRACT

Bus operations on arterials are often hindered by traffic signals and car queues. Transit signal priority (TSP) strategies can be used to improve bus operations on arterials. Analytically quantifying the impacts of TSP in mixed traffic, where cars and buses share lanes, is challenging since car queues can slow down buses, while slow-moving buses can create bottlenecks for cars. Furthermore, computational costs increase significantly when considering an arterial with multiple intersections. To tackle these challenges, this paper first develops a dynamic programming framework to model and evaluate TSP along an arterial. Next, the algorithm is utilized to determine the changes to car and bus delays as a result of TSP implementation along an arterial, and the sensitivity of the algorithm to the signal timing plan, bus stop locations and dwell durations, and the bus headway is evaluated. Next, a bi-level optimization framework is proposed to determine the optimal location of TSP implementation along arterials. Finally, the results of integrating TSP with dedicated bus lanes is evaluated. The results suggest that the benefits of TSP largely depend on the signal setting, and bus stop location, and that as the bus headway decreases, the marginal benefit of providing TSP also decreases. Additionally, the results suggest that the specific intersection at which TSP is implemented can play a large role on its operational impacts. Finally, it is found that in some scenarios, the benefits of implementing TSP alone can be larger than implementing dedicated bus lanes alone.

1. Introduction

Urban congestion is a problem that plagues many cities. Public transportation that can carry more passengers per vehicle than cars can help improve urban congestion. Public transportation (e.g., buses) is often seen as an inferior mode due to its slower travel speed. However, strategies that can help improve bus operations can be implemented along arterials to increase their travel speed. One popular strategy to increase bus travel speed is transit signal priority (TSP) that can allow buses to pass through signalized intersections faster. TSP changes the signal timing plan in a passive or active fashion to allow a bus to travel through the signalized intersection with less delay.

The benefits of implementing TSP along an arterial where cars and buses operate in mixed fashion are difficult to quantify due to the interactions between cars and buses. First, buses that travel on lanes mixed with cars often get delayed by car queues. Second, buses often travel slower than regular cars and hence can delay cars travelling behind them. To model these interactions, this paper utilizes

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the Lax-Hopf solution to the kinematic wave theory (Claudel and Bayen, 2010; Simoni and Claudel, 2017; Wu and Guler, 2019). A dynamic programming algorithm that can update signal settings with TSP implementation (Wu and Guler, 2019) is extended to determine the cumulative number of vehicles within a time–space plane along an arterial. This is then used to evaluate the bus and car delays along the arterial before and after TSP implementation.

This paper is the first to analytically study the impacts of TSP along an arterial considering mixed traffic operations (i.e., no dedicated bus lanes). The computational efficiency of the proposed analytical methodology allows to test the sensitivity of the algorithm to the signal settings, bus stop locations and bus dwell durations, and bus headways. Moreover, a methodology to identify critical intersections along an arterial where the implementation of TSP might be most beneficial is developed.

The remainder of the paper is organized as follows. In Section 2, a literature review is presented. Section 3 introduces the methodology followed by Section 4 where the results of a series of sensitivity analyses regarding the impacts of signal setting (cycle length and offset), the location of the bus stop, and bus headway are presented. In Section 5 a bi-level optimization framework is proposed to find the optimal location of TSP implementation along an arterial and is numerically evaluated. Section 6 shows the impacts of combining two common bus priority strategies – TSP and dedicated bus lanes. Finally, some concluding remarks on findings and future work are presented in Section 7.

2. Literature review

Many different ways for improving public transportation systems exist, including dedicated bus lanes (Levinson et al., 2002; Shalaby and Soberman, 1994; Surprenant-Legault and El-Geneidy, 2011; Ben-Dor et al., 2018), transit signal priority (Christofa et al., 2013a,b; Christofa and Skabardonis, 2011; Skabardonis, 2000), intermittent bus lanes (Chiabaut et al., 2012; Currie and Lai, 2008; Eichler and Daganzo, 2006; Guler and Cassidy, 2012; Viegas and Lu, 2004), or pre-signals (Guler et al., 2016; Guler and Menendez, 2014, 2015; Wu and Hounsell, 1998). Transit signal priority (TSP) is a strategy that can significantly benefit buses with minimal changes to the infrastructure. TSP can be deployed in passive, active or adaptive fashion (Christofa and Skabardonis, 2011; Smith et al., 2005). Passive TSP implementation involves developing signal timing and phasing plans offline to accommodate the travel speed of buses. Active TSP strategies rely on real-time information of bus location and can change the signal timing to allow the bus to pass the intersection with zero or little delay. Typical strategies used to provide active TSP include green extension, red truncation, phase swap or phase insertion (Christofa and Skabardonis, 2011; Skabardonis, 2000). Green extension extends the green time to allow a bus to pass at the end of the green time, while red truncation shortens the red time to allow the bus to depart from the intersection earlier. Phase swap changes the order of phases to allow for the bus phase to be activated sooner while a phase insertion inserts the bus phase whenever a bus would require it (Ekeila et al., 2009; Hu et al., 2014; Skabardonis, 2000). Adaptive strategies rely on real-time information from both cars and buses and aim at minimizing the delay for both modes simultaneously (Wu et al, 2017).

Studies have evaluated the benefits of TSP to buses, and potential negative impacts to cars using analytical methods or simulation. Simulation studies depend on large amounts of data to be created and validated (Radwan and Benevelli, 1983; Balke et al., 2000; Dion et al., 2004; Ngan et al., 2004). Additionally, it is difficult to generalize the observed trends since they are location specific. Hence, there is a need for analytical models to evaluate TSP implementation. Some analytical studies relied on delay functions that describe car delay as a function of density. For example, approximate equations of car delay, e.g., that can be found in the Highway Capacity Manual, were used in early studies to evaluate delay at an intersection with and without TSP (Jacobson and Sheffi, 1981; Sunkari et al., 1995).

There are a few studies that have investigated the impacts of TSP at an arterial level (Christofa et al., 2016; Stevanovic et al., 2008; Dion et al., 2004; Stevanovic et al., 2008; Wadjas and Furth, 2003; Zhenlong and Minghao, 2015; Chang et al., 2003). However, most of these studies either focus on signal timing optimization alone (Christofa et al., 2016; Stevanovic et al., 2008) or use simulations of case studies (Dion et al., 2004; Wadjas and Furth, 2003; Zhenlong and Minghao, 2015; Chang et al., 2003). One analytical study assessed the benefits and costs of TSP implementation along an arterial using a queuing theory based deterministic analytical model (Abdy Zeeshan R. and Hellinga Bruce R., 2011). This study found that TSP can reduce delay of cars and buses travelling along the arterial, and increase bus reliability, but also has the disadvantage of penalizing the cross-street traffic when high transit volumes exist at the corridor. A simulation study found that TSP increased the overall delay by 1.0% on the corridor, despite a bus delay reduction of 0.9% (Chang et al., 2003), however, another simulation study found that these negative impacts could be negligible during off-peak periods (Dion et al., 2004). Additionally, TSP could negatively impact the coordination of signals along an arterial, and strategies that specifically aim at maintaining a green-wave can reduce the car delay at downstream intersection by up to 10% (Zhenlong and Minghao, 2015).

Other work have looked at the optimal location of dedicated bus lanes (Bayrak and Guler, 2018; Chen, 2015; Khoo et al., 2014; Mesbah et al., 2011, 2008), and bus detectors (Liu et al., 2004; Zhou et al., 2006) along a corridor using analytical tools or a microsimulation. To the authors' knowledge, only three studies have considered the optimal location of TSP implementation (Bagherian et al., 2014, Wu and Guler, 2018, Bayrak and Guler, 2020). But those studies either used variational theory with the assumption of a dedicated bus lane to separate the buses and cars, such that there are no interactions between the two modes (Wu and Guler, 2018), or used microsimulation such that the results were specific to the microsimulation method used (Bagherian et al., 2014).

While other studies have considered the impacts of TSP along an arterial, there is no analytical work that considers mixed bus and car operations taking into consideration the low travel speed of buses. Multiple studies have shown that the speed of buses is smaller than cars in urban arterials, even without considering stops (Liu et al., 2015; Castrillon and Laval, 2018; Du et al., 2017; Nagatani, 2016). Some work that used kinematic wave theory to study impacts of TSP either assumed that buses have their own lanes (Truong et al., 2017; Wu and Guler, 2018), or that buses and cars drive at the same speed (Sunkari et al., 1995). The solution to the LWR problem quantifies the vehicle densities on a roadway section and allows for slow moving buses to be modelled as moving bottlenecks (Daganzo and Laval, 2005; Munoz and Daganzo, 2002) such that cars incur additional delays. Some work that used kinematic wave theory to study impacts of TSP either assumed that buses have their own lanes (Abdy and Hellinga, 2011), or that buses and cars drive at the same speed (Sunkari et al., 1995). One solution method to the kinematic wave theory, named variational theory, discretizes space and time to solve for the capacity of arterials (Daganzo, 2005; Daganzo and Menendez, 2005). A recent work used this solution method to estimate the impacts of TSP along an arterial assuming dedicated bus lanes (Wu and Guler, 2018). Another solution method to the kinematic wave theory is to apply the Hamilton-Jacobi partial differential equation (PDE). The solution of this PDE can also be cumbersome and computationally inefficient. One approach to solving this equation is also to discretize it into nodes and links and consider flows of vehicles, as is done in (Chow and Li, 2019) to study multimodal corridors. Another solution method is to use the Lax-Hopf equation to compute cumulative vehicle numbers (Claudel and Bayen, 2010) and quantify delay using queueing theory (Makigami et al., 1971). The Lax-Hopf solution can account for more flexible bus operations since there is no restriction on the solution space and makes it easier to incorporate moving bottlenecks (Mazare et al., 2011). Additionally, it is computationally efficient allowing for larger time–space domains to be analyzed. A recent study has used this approach to analyze the impacts of TSP at a mixed-modal intersection (Wu and Guler, 2019). However, mixed-modal bus operations have not been analyzed at the arterial level.

3. Methodology

The Lax-Hopf equation is used as the methodology to evaluate car operations along the arterial. In this section, we briefly summarize the main features of the generalized Lax-Hopf equation (Simoni and Claudel, 2017; Mazare et al., 2011; Aubin et al., 2008). The Lax-Hopf equation is a solution method to the LWR problem (Lighthill and Whitham, 1955; Richards, 1956) that assumes a homogenous roadway segment, within the time–space domain $\boldsymbol{D}$ of length L and time duration T, on which vehicle operations can be described by a triangular fundamental diagram with free-flow speed v_f, backward wave speed $-w$, capacity vehicle flow q_c, and density at capacity, k_c, and jammed density k_J. The Lax-Hopf equation quantifies the cumulative vehicle numbers, denoted by $N(\mathrm{t},\mathrm{x})$, within the domain of interest using a set of boundary conditions, see Eqs. (1) and (2) (Daganzo, 2005).

$$N(\mathrm{t},\mathrm{x}) = \inf\{C(t-\Delta T, x-u\Delta T) + \Delta T R(u)\} \tag{1}$$

$$s.t. u \in [-w, v_f], \text{and} (t-\Delta T, x-u\Delta T) \in \boldsymbol{D} \tag{2}$$

where $C(t,x)$ corresponds to the cumulative count of numbers associated with a set of boundary conditions, and $R(u)$ represents a cost function associated with a fundamental diagram. For a triangular fundamental diagram $R(u)$ is equivalent to the maximum rate that a vehicle travelling at speed u could pass a path, which can be expressed as follows:

$$\mathrm{R(u)} = \begin{cases} k_c(v_f+w) & u=-w \\ k_c v_f & u=0 \\ 0 & u=v_f \end{cases} \tag{3}$$

Denote the cumulative vehicle count at the beginning of condition i as N_i, the location at the beginning of condition i as x_i, the time at the beginning of condition i as t_i, the density at point (t,x) as k, the flow at point (t,x) as q. The boundary conditions,$C(t,x)$, account for:

(1) The initial conditions $C_{ini}(t,x)$, i.e., the predicted cumulative vehicle count at point (t,x) based on cumulative vehicle count along the arterial at time $t=0$.

If $k \in [0,k_c]$, it imposes a free-flow state, hence,

$$C_{ini}^i(t,x) = \begin{cases} N_i + k\big(x-(x_i+v_f t)\big) & x \in [x_i+v_f t, x_{i+1}+v_f t) \\ N_i + k_c\big(x-(x_i+v_f t)\big) & x \in [x_i - wt, x_i + v_f t] \end{cases} \tag{4}$$

Else if $k \in (k_c, k_J]$, this imposes a congested state,

$$C_{ini}^i(t,x) = \begin{cases} N_i + k_J wt - k(x-(x_i-wt)) & x \in [x_i - wt, x_{i+1} - wt) \\ N_{i+1} - k_c\big(x-(x_{i+1}+v_f t)\big) & x \in [x_{i+1}-wt, x_{i+1}+v_f t] \end{cases} \tag{5}$$

The final $C_{ini}(t,x)$ can be determined as the minimum of the free-flow and congested states.

$$C_{ini}(t,x) = \min_i\{C_{ini}^i(t,x)\} \tag{6}$$

(2) The upstream conditions $C_{up}(t,x)$, i.e., the predicted cumulative vehicle count at point (t,x) based on the maximum possible vehicle counts at the upstream ends of an arterial (e.g., due to demand).

$$C_{up}^{i}(t,x) = \begin{cases} N_i + q\left(t - t_i - \frac{x-0}{v_f}\right) & t \in \left[t_i + \frac{x-0}{v_f}, t_{i+1} + \frac{x-0}{v_f}\right) \\ N_{i+1} + q_c\left(t - t_{i+1} - \frac{x-0}{v_f}\right) & t \in \left[t_{i+1} + \frac{x-0}{v_f}, T\right] \end{cases} \tag{7}$$

$$C_{up}(t,x) = \min_i \left\{ C_{up}^{i}(t,x) \right\} \tag{8}$$

(3) The downstream conditions $C_{down}(t,x)$, i.e., the predicted cumulative vehicle count at point (t,x) based on the maximum possible vehicle counts at the downstream end of an arterial (e.g., due to signals),

$$C_{down}^{i}(t,x) = \begin{cases} N_i + k_J(X-x) + q\left(t - t_i - \frac{X-x}{w}\right) & t \in \left[t_i + \frac{X-x}{w}, t_{i+1} + \frac{X-x}{w}\right) \\ N_{i+1} + k_J(X-x) + q_c\left(t - t_{i+1} - \frac{X-x}{w}\right) & t \in \left[t_{i+1} + \frac{X-x}{v_f}, T\right] \end{cases} \tag{9}$$

$$C_{down}(t,x) = \min_i \left\{ C_{down}^{i}(t,x) \right\} \tag{10}$$

(4) The internal conditions $C_{int}(t,x)$, i.e., the predicted cumulative vehicle count at point (t,x) based on the maximum passing rates of moving bottlenecks.

$$C_{int}^{i}(t,x) = N_i + q_r(t - t^{'} + t_i) + k\_cv't' \tag{11}$$

$$C_{int}(t,x) = \min_i \left\{ C_{int}^{i}(t,x) \right\} \tag{12}$$

where, $t^{'}, v'$ are given by the follow equations for the three domains.

(a). When (t,x) within forward domain,

$$t^{'} = \frac{x - (x_i + v_b(t-t_i))}{v_{f-}v_b}, v^{'} = 0 \tag{13}$$

(b). When (t,x) within backward domain,

$$t^{'} = \frac{(x_i + v_b(t-t_i)) - x}{v_b + w}, v^{'} = v_f + w \tag{14}$$

(c). When (t,x) within central domain,

$$t^{'} = t - t_{i+1}, v^{'} = v_f - \frac{x - x_{i+1}}{t^{'}} \tag{15}$$

Then, the cumulative vehicle number, $N(\mathrm{t},\mathrm{x})$, is then determined as the minimum of the four boundary conditions, see Wu and

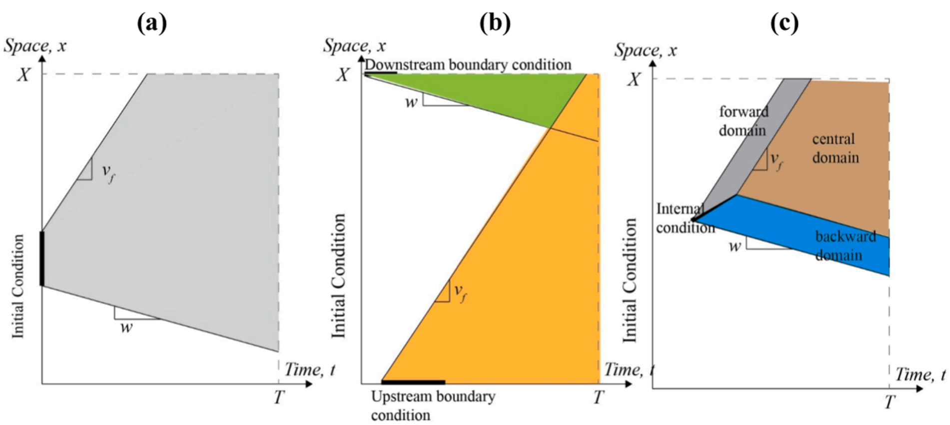

Fig. 1. Illustration of initial, boundary and internal conditions (Wu and Guler, 2019).

Guler. 2019 for more details. Fig. 1a and 1b shows examples of the domains of influence of the initial condition, and upstream and downstream conditions, respectively. The initial condition is the set of densities, k, over space, x, at time$t = 0$, This condition either propagates a free-flow state downstream at free flow speed v_f, if $k_i \in [0, k_c]$, or a congested state upstream at the backward wave speed, w, if $k_i \in [k_c, k_J]$, see Fig. 1a. The upstream condition is a set of flows, q, over time,t at the upstream location, x_0. These conditions are propagated downstream at free flow speed v_f, see Fig. 1b. On the other hand, the downstream conditions are a set of allowable maximum flows, q_{max}, over time,t at the downstream location, x_L.The downstream conditions are propagated upstream at the backward wave speed, w, see Fig. 1b.

Table 1
Illustration of the methodology.

Inputs

- *Fundamental diagram (FD) parameters*
- ▪ Free-flow and backward wave speed ($v_f, -w$), capacity and jammed densities (k_C, k_J)
- *Demand*
- ▪ Initial densities along the arterial (k_{ini}), upstream demand (q)
- *Bus parameters*
- ▪ The time when the bus enters the arterial (t_a), the maximum bus speed (v_m)
- ▪ Bus stop locations ($L_s = [l_{s1}, l_{s2}, \cdots]$), bus dwell durations ($S = [s_1, s_2, \cdots]$),
- ▪ Capacity of roadway when a bus dwells (q_b), maximum passing rate of a bus (q_r)
- *Signalized intersection parameters*
- ▪ Intersection locations ($L = [l_1, l_2, \cdots]$), TSP bus detector locations ($L_d = [l_{d1}, l_{d2}, \cdots]$)
- ▪ Traffic signal: cycle length, green ratio, offset (cl, g, o)
- ▪ Maximum TSP time change (dt_m)
- *Computation parameters*
- ▪ Time and space domain (T, X), Time and space steps ($\Delta t, \Delta x$), total number of time steps, m and total number of space steps,n

Initialization

- *Initial condition*
- ▪ Compute the vehicle number within the whole computation domain with initial densities, i.e., vehicle count matrix$N^{m\times n}$
- *Boundary condition*
- ▪ Update vehicle count matrix $N^{m\times n}$ with upstream and downstream boundary conditions
- *Bus trajectory*
- ▪ Generate list $\boldsymbol{Traj}$ with first row: $[t_b, x_b, v_b]$,$t_0 = t_a$,$x_0 = 0$,$v_0 = v_m$, where t_b is time at step b of bus movement, x_b is location of bus at step b, and v_b is speed of bus at time step$b$
- *Intersection signal*
- ▪ Generate list $\boldsymbol{Sig}$ to record beginning and ending times of red phases, r_{ij}^a and r_{ij}^e, respectively, for each intersection i and red phase$j$

Incorporate bus movement and TSP

- *Track bus movement*
- ▪ Update the bus trajectory list $\boldsymbol{Traj}$ by time step, i.e.,$t_b = t_{b-1} + \Delta t$
- ▪ If bus:
  - o Travels at its free flow speed acting as an active bottleneck, update $N^{m\times n}$ with internal condition: bus moving bottleneck
  - o Enters congested traffic region and cannot travel at its free flow speed, set v_b to surrounding traffic speed,v_s
  - o Arrives at a bus stop i, update $N^{m\times n}$ with internal condition: bus stationary bottleneck
  - o Arrives at TSP detector location, record the time t_b and call for TSP signal timing update
- ▪ If signal is red at time t_b, update $N^{m\times n}$ with internal condition: signal stationary bottleneck
- *TSP update*
- ▪ Update signal timing if TSP is activated at intersection i,
- o Estimate bus arrival time t_{xi} at intersection i assuming bus travels at speedv_m
- o Find the intersection i and red phase j, r_{ij} that would be changed due to TSP
- o If $t_{xi} \le r_{ij}^a + dt_m$, green extension is called and$r_{ij}^a = t_{xi}$
- o If $r_{ij}^a + dt_m < t_{xi} \le r_{ij}^e$, red truncation is called and$r_{ij}^e = \max(r_{ij}^e - t_{xi}, r_{ij}^e - dt_m)$

Post-processing & output

- *Change in bus travel time*
- ▪ Compute the bus delay
- ▪ Compute the difference in the times when bus exits the arterial, without TSP and with TSP,T_B
- *Change in arterial car delay*
- ▪ Compute the car delay
- ▪ Following queuing theory (Makigami et al., 1971; Wada et al., 2017), the overall change in cumulative car delay due to TSP, T_c is calculated as $T_c = \sum_{t=0}^{T} N_T(t, X) - N(t, X)$ where, $N_T(t, X)$ and $N(t, X)$ denotes the vehicle numbers at arterial exit location at time t, with TSP and no TSP, respectively
- *Change in cross-street car delay*
- ▪ Following queuing theory, compute the change in car delay due to change in signal setting

Notice that the initial conditions, and upstream and downstream boundary conditions are all inputs to the algorithm. The upstream conditions can be used to input a time varying demand onto the roadway, while the downstream conditions can be used to input traffic signals at the downstream end of a roadway.

Finally, the internal conditions represent the presence of a moving and/or stationary bottleneck within the study area. These internal conditions are a measure of the cumulative vehicle numbers along a bottleneck's trajectory. For example, for a moving bottleneck, the bottleneck capacity is derived from the fundamental diagram as in Eq. (16) (Munoz and Daganzo, 2002).

$$q_r = k_C\left(v_f - v_b\right)\frac{n_l - 1}{n_l} \tag{16}$$

where v_b denotes the bottleneck velocity and n_l denotes the number of lanes.

For a stationary bottleneck, the bottleneck capacity can be calculated as in Eq. (17) (Munoz and Daganzo, 2002). Note that there are no bus bays at bus stops, buses completely block the outside lane when stopped to board/alight passengers, hence the capacity of a stationary bottleneck imposed by a bus dwelling at bus stops is q_b, see Eq. (17).

$$q_b = q_c\frac{n_l - 1}{n_l} \tag{17}$$

The cumulative counts measured at these internal locations can be propagated within the forward, backward and central domains, see Fig. 1c. The forward and central domains remain in a free-flow state while the backward domain becomes congested. More details on the solution method can be found in (Simoni and Claudel, 2017; Wu and Guler, 2019).

While the starting point and speed of a moving internal condition is an input to the model, the exact trajectory of these moving bottlenecks, e.g., buses, is calculated within the model. Hence, buses that get stuck in car queues are accurately accounted for. A pseudocode for how the bus movement is tracked within the algorithm is shown in Table 1. On the other hand, stationary internal conditions can be used to input dynamic signal operations into the model. While this is initially an input, the signal timing will depend on whether transit signal priority is provided or not. Hence, next the algorithm to incorporate and evaluate TSP in the Lax-Hopf framework along an arterial is discussed and shown in Table 1.

The goal of the TSP algorithm is to calculate the changes in bus travel times and the changes in arterial and cross-streets car delay after implementing TSP at multiple intersections. The bus is modelled as a moving bottleneck, and on-street bus stops are included as temporary stationary bottlenecks. Two common TSP options are considered – green extension and red truncation, assuming the maximum variation in signal timing is Δt_m. The maximum variation is set such that a minimum green time is maintained for the cross-street. Note that demonstrated herein are two common conventional TSP strategies, however, the proposed methodology can easily be extended to incorporate other TSP treatments, such as phase skipping, phase rotation and phase reallocation.

An arterial of length X is analysed for a duration of time, T. The structure of the algorithm is illustrated in Table 1. It mainly consists of four parts.

Part 1, Inputs. A set of basic parameters is inputted into the model, including the fundamental diagram parameters, the initial conditions, as described by Eqs. (4)–(6), the upstream conditions as described by Eqs. (7) and (8), and the downstream conditions as described by Eqs. (9) and (10).

Part 2, Initialization. A vehicle count matrix N is initialized with initial and boundary conditions. The elements of N represent the cumulative vehicle count at a specific point of time and location. Additionally, the beginning of each bus trajectory is initialized, and the initial (no TSP) signal timing schedule is set.

Part 3, Incorporate Bus Movement and TSP. The numerical scheme is then used to simulate bus movement along the arterial. A bus travelling at its maximum speed, v_m ($v_m < v_f$), starts travelling at the speed of the surrounding traffic, v_s, when traffic becomes congested and stops at the bus stop or at red traffic lights. When a bus arrives at a TSP detector, the arrival time of the bus to the main signal is predicted assuming the bus can travel at maximum speed, v_m. Then, the *TSP update* algorithm changes the signal setting to allow the bus to clear the intersection as soon as possible. A green extension (GE) is activated if the predicted bus arrival time is within Δt_m(maximum allowable time change to signal setting) of the signal turning red. Red truncation (RT) is activated for all remaining buses whose predicted arrival times fall within the red duration. The *TSP update* algorithm runs as a dynamic program where each intersection acts as a state since there are multiple intersections and the results of TSP activation at one intersection can impact the downstream intersections.

Part 4, Post-processing & Output. To quantify the overall impacts of TSP implementation, a baseline case where TSP is not implemented is also considered.

The bus delay is calculated as the difference in the departure time of a bus from the downstream end of the arterial, and the departure time that the bus would have experienced given there were no traffic signals or other cars (i.e., bus dwell at stops is not considered part of the delay).

The car delay is computed using queueing theory (Makigami et al., 1971; Wada et al., 2017), as the difference in the cumulative vehicle count at the downstream end of the arterial, $N(t,X)$, to a virtual cumulative vehicle count at the downstream of the arterial if each vehicle could travel at free flow. For vehicles that do not traverse the entire arterial, the downstream is defined as the location where they are at the end of the time period. Only the part of the vehicles trajectory that is within the analysis window is used to calculate the delay. Additionally, the cross-street delay (and change in cross-street car delay) at every intersection is calculated using queueing theory based on the changes in signal setting.

Finally, the bus delay, the arterial car delay and the cross-street car delay are outputted from the algorithm considering both no TSP and TSP scenarios. The arterial car delay is a summation of delays experienced by cars traveling both in the same and opposite direction as the bus and represents the cumulative delay of each car from intersection 1 through intersection 7. The arterial car delay and the cross-street car delay are presented independently. This is done since after the first intersection, vehicle arrivals to consecutive intersections are not random, rather they are platooned. Hence, the delay is significantly different along the arterial than at the cross-street where each individual cross-street experience random arrivals.

4. Numerical results and sensitivity analysis

The algorithm presented in Table 1 is used for numerical simulations of arterials. A baseline scenario is first setup to verify the accuracy and efficiency of the model, including the TSP implementation. Then the sensitivity of bus and car delays after TSP implementation to signal setting (cycle length and offset), bus stop location (near-side, mid-block, and far-side) and dwell duration, and bus headway are tested.

4.1. Baseline case setup and verification

A 7-intersection arterial of length $X = 1300m$ is considered for analysis. It is assumed that there are no turning movements to simplify the analysis since turning movements would create discontinuities in time and space in the propagation of vehicles. Turning inflows and outflows at intersections could be incorporated by modeling them as "internal conditions" or decomposing the arterial into sub-arterials separated by intersections where main turning movements occur.

The intersections are labelled 1 through 7, with 1 denotes the upstream most intersection from which the bus and cars enter the arterial. The number of lanes in each direction is $n_l = 2$. The locations (in meters) of signalized intersections are $\boldsymbol{L} = \{200, 350, 500, 650, 800, 950, 1100\}$, i.e., all of the signalized intersections are 150 m apart. While the spacing of intersections can easily be changed in this analysis, previous work have shown that the block length divided by the cycle length can be used as a dimensionless parameter when analyzing network operations. Hence, the sensitivity analysis on cycle length in this paper can also be interpreted as a sensitivity analysis on block length (Laval and Castrillon, 2015; Gayah et al., 2014). Traffic operations on this roadway is represented with a triangular fundamental diagram with a maximum flow rate, $q_C = 0.75veh/sec$, free-flow speed $v_f = 15m/sec$, backward wave speed $-w = -5m/sec$, and jam density $k_J = 0.2veh/m$. It is assumed that buses travel slower than cars with a speed, $v_m = 10m/sec$, creating a moving bottleneck with a maximum passing rate, $q_r = 0.125veh/sec$, see Eq. (16), and a stationary bottleneck at bus stops with a maximum passing rate, $q_b = 0.375veh/sec$, see Eq. (17). Notice that a relatively low bus speed is used in this paper to illustrate the methodology in line with other similar work (Liu et al., 2015). The methodology can easily be applied to different bus speeds.

A common cycle length and zero offset along the arterial are assumed, unless otherwise specified (Girault et al., 2016). The car demand for the arterial (both bus travel direction and opposite direction) and cross-streets are equal and denoted, $q^{'} = q = 0.3veh/sec$, respectively. A green ratio of all intersections is determined as $g = \frac{q}{q^{'}+q} = 0.5$ to fairly assign the green time between conflicting directions (i.e., cross-street and arterial street). Buses are assumed to arrive every 5 min, bus stops are placed mid-block at $\boldsymbol{x}_s = \{260, 550, 850, 1150\}m$, and buses are assumed to dwell for a duration of $S = 30\text{sec}$ at each stop, unless otherwise specified. While in this work only one bus per cycle is assumed to arrive, the framework of evaluating the impacts of TSP is flexible enough to account for more than one bus per cycle, or buses that arrive from multiple directions. The analysis of more than one buses per cycle or buses arriving from multiple directions would simply require updating the algorithm to trigger TSP using one of the strategies proposed in the literature to resolve conflicting TSP requests (e.g., Christofa and Skabardonis, 2010; He et al., 2014). The sensitivity of the results to cycle length, offset, bus stop location, bus dwell duration, randomness in bus dwell time, and bus headway are evaluated in the following subsections.

The bus detector location $x_d = 60m$ is chosen to ensure that the detector is placed downstream of the bus stop, detecting buses only after their departure from the bus stop. This bus detector location was shown to result in the largest delay savings for an isolated intersection (Wu and Guler, 2019). Two common TSP treatments – green extension (GE) and red truncation (RT) are considered, where a maximum change in signal timing of $\Delta t_m = 5sec$is allowed.

The analysis window is chosen considering two factors. First, the analysis window needs to be large enough to accommodate many signal cycles such that different scenarios of TSP trigger are realized. This allows for the average delay calculated in the analysis window to approach the long-term value. Second, the computational time increases as the analysis window increases. Considering both of these constraints, an analysis window of $\boldsymbol{T} = 800sec$ and $\boldsymbol{X} = 1300$ m is chosen to balance between the computational efficiency and accuracy. The cycle during which the first bus arrives is deemed as the first cycle. The expected bus travel time savings and changes in car delay are shown as an average of all possible uniformly distributed bus arrival times across the first cycle at intersection 1. Notice that while multiple different bus arrival times are tested, the output of each possible bus arrival time is deterministic, hence, all the results are reproducible (which is not true for simulation studies). The proposed method is implemented using Python 3, in Google Colaboratory[1].

For the baseline case with 120 sec cycle length, 0 sec offset, 5 min bus headway, example trajectories of car and buses along the arterials when TSP is not implemented, and when TSP is implemented are shown in the time–space diagrams as in Fig. 2a and 2b, respectively. In these figures, the red solid lines denote the red phase of each cycle, the red dashed lines denote a red phase with

[1] The original code can be found here: https://colab.research.google.com/drive/1D7bj5BKdV1_QTWrdhbQm07D7zpTaQ6-C.

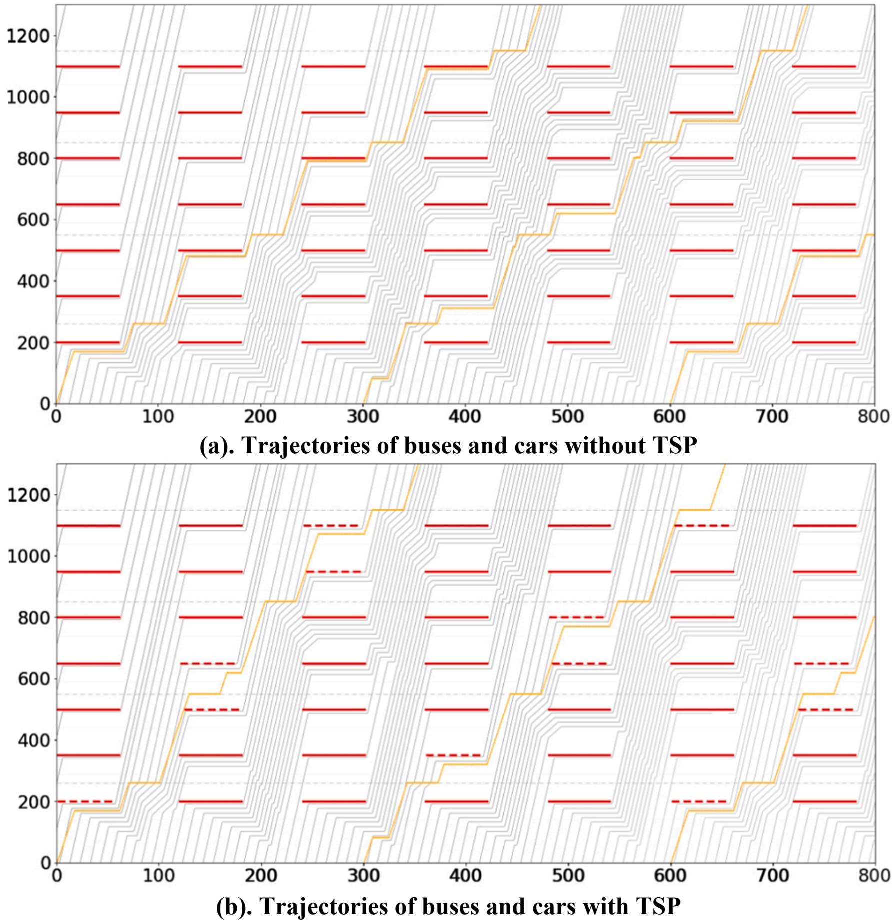

Fig. 2. Time-space diagram of baseline case.

triggered TSP strategy, the orange lines represent the trajectories of buses and the gray dashed horizontal lines represent the location of bus stops. By comparing Fig. 2b to 2a, it can be seen that both buses that completely traverse the arterial benefit from TSP and are able to depart earlier.

Based on this baseline case, the delay of cars and buses along the arterial with and without TSP are calculated as Table 2. Note that as mentioned earlier, these results are obtained by considering all possible arrival times of buses to the first intersection and taking the average of all the results. As can be seen, both buses and arterial cars benefit from TSP in this baseline scenario. Results for the cross-street cars are not shown here since the methodology to calculate those have been well established in the literature.

We further analyze the distribution of total delay amongst the 7 intersections for the case where TSP is implemented as shown in Fig. 3. This figure shows that both the cars and buses encounter the largest delays at the first intersection. This result is expected since the arrival rate of each vehicle to the first intersection is uniformly distributed. After intersection 1, the traffic flow is platooned, thus the delay decreases. In addition, because the buses reach Intersection 1 evenly, the chance of implementing RT is much higher than GE. However, at other intersections there are different distributions of the two TSP strategies.

A microsimulation is done using Aimsun Next to validate the proposed method. The same baseline scenario with and without TSP is simulated in the microsimulation, and the resulting vehicle delays from the simulation are shown in Table 3. It can be seen that the microscopic simulation results in slightly larger average delays than the analytical method proposed in this paper due to the randomness in arrivals. Furthermore, in both methods the bus delay is larger than the car delay, and while TSP benefits both arterial cars and buses, benefits of TSP to buses is larger than to cars. Hence, the results of the analytical method and microscopic simulation are consistent.

4.2. Sensitivity to signal setting

The sensitivity of the algorithm to the signal setting is analyzed for different cycle lengths, and different offsets in two consecutive subsections.

Table 2
Results of baseline case.

Indicators	Total delay sec/km	Delay per vehicle sec/(veh*km)
Bus delay without TSP	715.29	163.84
Bus delay with TSP	257.48	94.40
Bus delay benefit from TSP	457.81	69.44
Arterial car delay without TSP	27188.09	108.92
Arterial car delay with TSP	23074.25	90.19
Arterial car delay benefit from TSP	4113.84	18.73

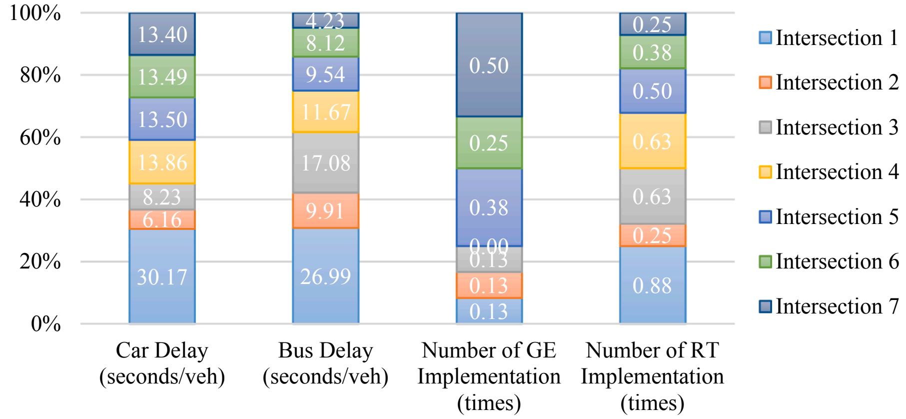

Fig. 3. Distribution of delay and TSP strategies across intersections.

Table 3
Delay from microsimulation.

Indicators	Delay sec/(veh*km)	Standard deviation sec/(veh*km)
Bus delay without TSP	160.31	29.87
Bus delay with TSP	106.91	32.43
Bus delay benefit from TSP	53.40	–
Arterial car delay without TSP	107.16	24.60
Arterial car delay with TSP	101.72	28.13
Arterial car delay benefit from TSP	5.44	–

4.2.1. *Cycle length*

In this experiment it is assumed that: (1) buses do not stop at any bus stops, and (2) all 7 intersections are equipped with TSP capability. Note that the presence of bus stops would only change the average speed of the bus, and hence would not impact the general trends observed. Different common cycle lengths, $\boldsymbol{C} = \{50, 60, 70, 80, 90, 100, 110, 120\}$ sec, are considered, assuming all signals along the arterial turn green simultaneously, i.e., the offset is zero. The green ratio, g, is kept constant at 0.5.

The change in bus travel time is shown in Fig. 4a as a benefit, i.e., reduced bus delay. As can be seen, TSP always reduces bus delay, however, the magnitude depends on the cycle length. Additionally, average bus delay is reduced more by providing red truncation (RT) than green extension (GE). This is due to the fact that GE relies on buses arriving at the intersection within a 5 sec window after the signal turns red and, hence is not activated frequently along the corridor. On the other hand, RT is activated for any bus that arrives during the red period that cannot receive GE. Hence, even though the bus delay saving is large with GE, not many buses receive this priority. The changes in arterial and cross-street car delays due to TSP under different common cycle lengths are exhibited in Fig. 4b and c, respectively. Notice that in these figures positive values indicate benefits, i.e., reduced car delay. As can be seen, impacts of TSP varies among different cycle lengths, and there is an obvious relationship between TSP impacts and cycle length. The reduction in arterial car delay is generally positively correlated with reduction in bus travel times, despite some exceptions. This is because cars that travel parallel with the bus also benefit from TSP. As for the cross-streets TSP increases the car delay, as expected.

Notice that these results are not presented to help choose the cycle length, since they depend largely on the scenario considered, i.e., the distance between intersections, offset selection, etc. However, they do provide the insight that the choice of cycle length can significantly impact how TSP changes bus and car delay. Moreover, it is possible to identify cycle lengths where large delay savings for buses and cars are observed, with smaller increase in delay to cross-streets, e.g., a 60 sec cycle length in this example. On the other hand, there are some cycle lengths that don't benefit buses, don't reduce arterial car delays much while increasing cross-street delays significantly, e.g., 110 sec. Hence, these results are cautionary to the application of TSP along arterials in that the choice of cycle length

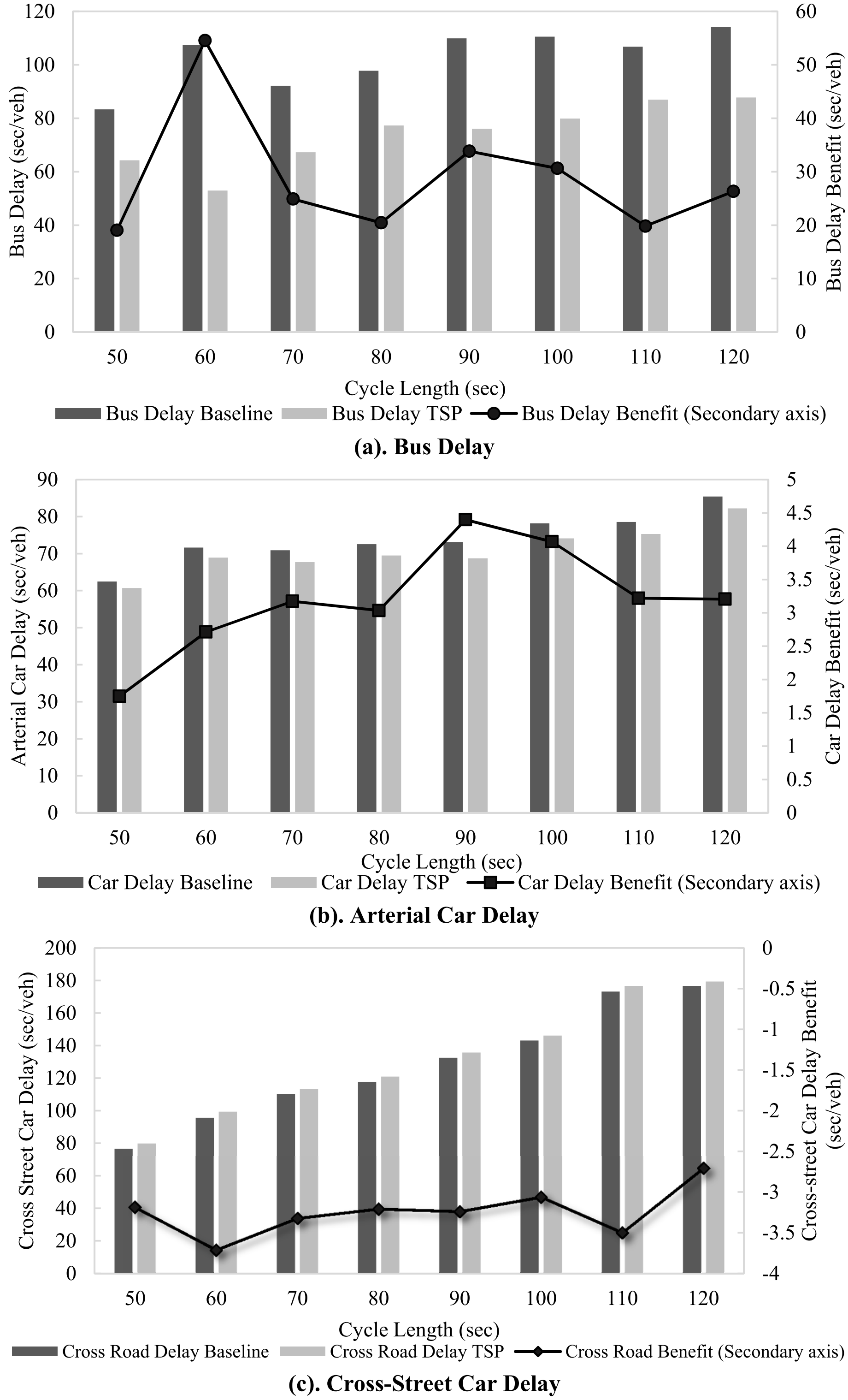

Fig. 4. Sensitivity tests of cycle length.

can make a significant difference in how TSP can reduce bus and arterial car delays and increase cross-street delays. The bus travel time savings and changes in car delay are shown as an average of all possible uniformly distributed bus arrival times across the first cycle at intersection 1. Notice that delay savings themselves vary with cycle length. This is mainly due to the fact that the bus travel time savings and changes in car delay are shown as an average of all possible uniformly distributed bus arrival times across the first cycle at intersection 1. Hence, the delay savings depend heavily on whether buses arrive to each intersection at the “right” time to receive priority or not.

The results of additional tests with buses stopping at bus stops with 30 sec dwell time are shown in Fig. 5. Looking at this figure, the conclusions on RT saving more delay than GE and the cycle length significantly impacting how TSP changes bus and car delay remain the same. The specific trends with respect to the cycle length are no longer valid since the average bus travel speed is impacted by the bus stops, leading to different trigger times of the red truncation and green extension.

4.2.2. Offset

In this experiment, four different types of signal offsets along the arterial in the bus movement direction are considered: (1) zero offset; (2) 10 s offset (green wave offset, block length divided by free-flow speed, v_f); (3) 15 s offset (block length divided by bus maximum speed, v_m); and (4) bus favorable offset (block length divided by bus maximum speed plus the dwell time at bus stops). Note that for scenarios 4 and 9 the offset depends on whether there is a bus stop between intersections or not, i.e., if there is a bus stop between intersections the offset value is 45, where as if there is no bus stop between intersections the offset value is 15. Also, short (60*sec*) and long (120*sec*) common cycle lengths are considered. Hence, there are a total of 6 cases:

(1) zero offset: $\mathbf{O} = [0,0,0,0,0,0]$**;** common cycle length: 60***sec***
(2) 10 sec offset:$\mathbf{O} = [10,20,30,40,50,60]$; common cycle length: 60***sec***
(3) 15 sec offset: $\mathbf{O} = [15,30,45,60,75,90]$; common cycle length: 60***sec***
(4) bus favorable offset: $\mathbf{O} = [45,60,105,120,165,180]$; common cycle length: 60***sec***
(5) zero offset: $\mathbf{O} = [0,0,0,0,0,0]$; common cycle length: 120***sec***
(6) 10 sec offset:$\mathbf{O} = [10,20,30,40,50,60]$; common cycle length: 120***sec***
(7) 15 sec offset: $\mathbf{O} = [15,30,45,60,75,90]$; common cycle length: 120***sec***
(8) bus favorable offset: $\mathbf{O} = [45,60,105,120,165,180]$; common cycle length: 120***sec***

Additionally, two scenarios for each case where buses are assumed not to dwell at any bus stops or assumed to dwell at mid-block bus stops for 30 s are considered. The bus delay, arterial car delay and cross-streets car delay for all considered cases as well as the benefits of TSP are exhibited in Fig. 6a, b, and c, respectively. For the zero-offset case (cases 1 and 5 in Fig. 6), both buses and cars traveling along the arterial experience frequent stops due to poor signal coordination regardless of whether buses dwell at stops or not. Hence, TSP activation brings significant benefits to buses and arterial street cars, in effect acting as coordination. Correspondingly, there is a considerable increase in cross-street cumulative car delay. It’s worth noting that when buses dwell at stops, the zero-offset becomes favorable to buses and results in low bus delays. This is only an artifact of the bus dwell times and cycle lengths syncing up due to the specific parameters used, and is not expected to be a generalizable result.

For the 10 sec offset cases (cases 2 and 6 in Fig. 6), when buses don’t stop at bus stops, most cars experience only one stop along the entire arterial. Buses also benefit from the reduced car queues and experience only small delays. In this scenario, the absolute delay of cars is the lowest and the benefit of TSP to arterial cars and buses, and the costs to cross-street cars decreases compared to the zero-offset scenario.

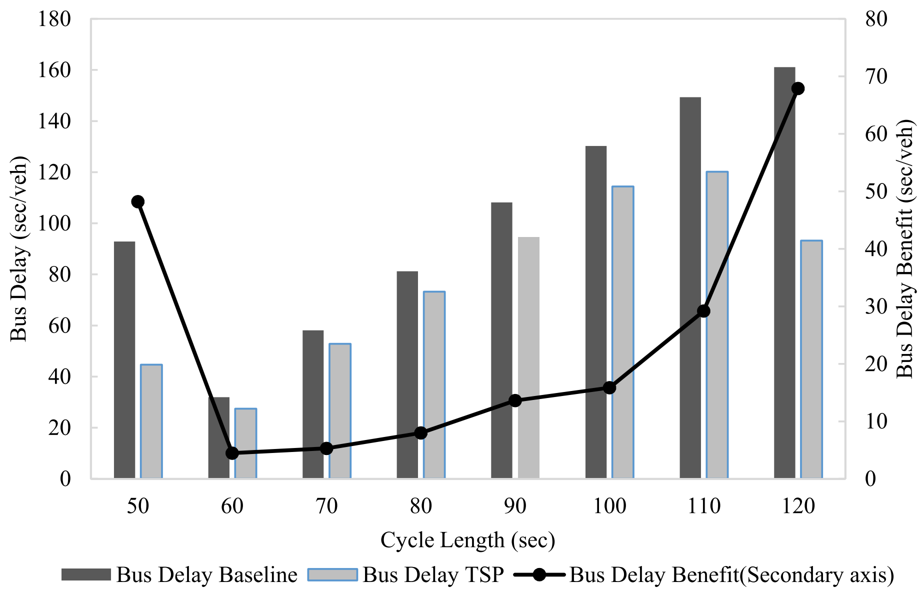

Fig. 5. Bus delay with a 30 sec dwell time at bus stops.

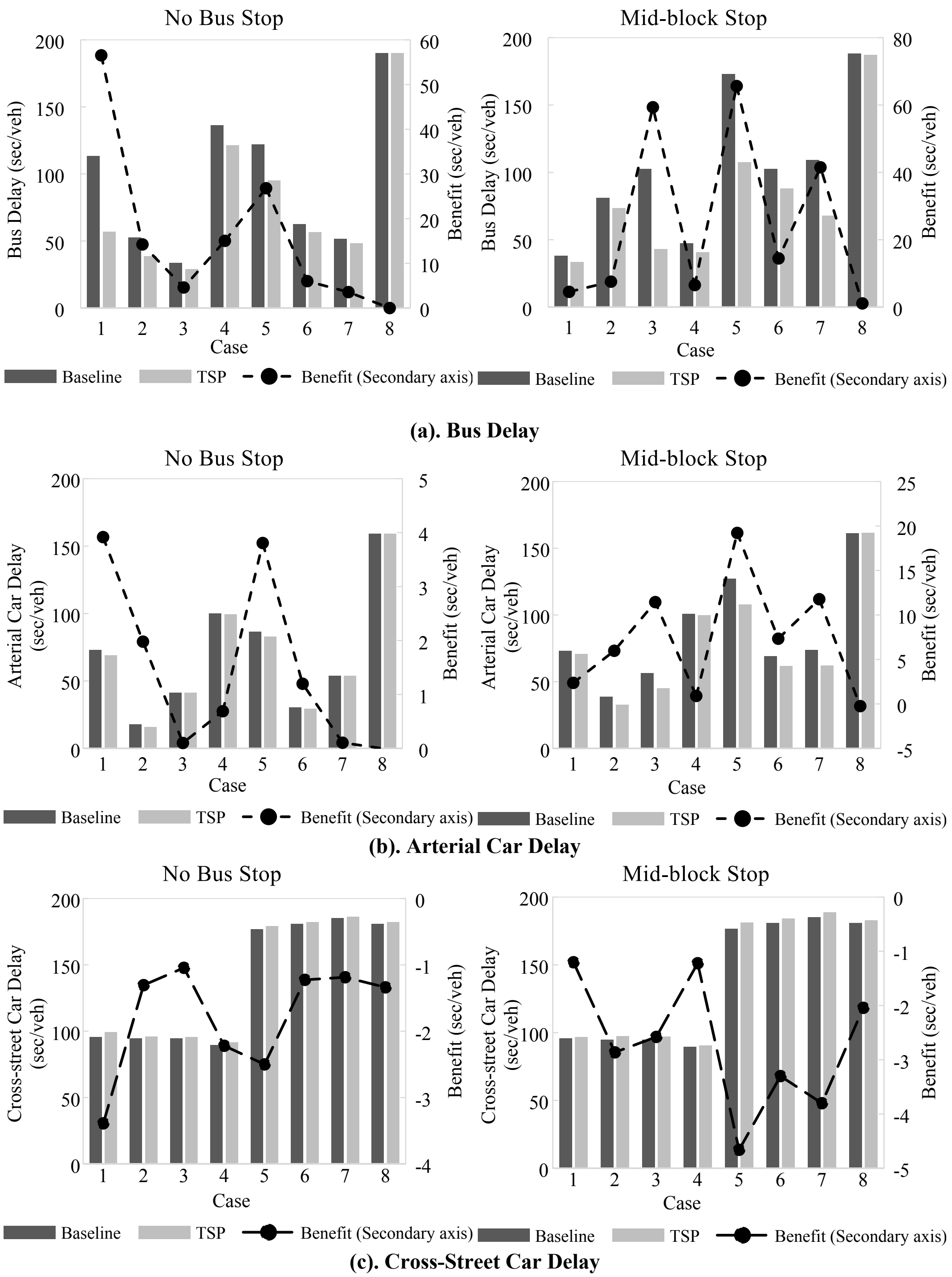

Fig. 6. Sensitivity analysis of offsets.

The 15 sec offset shown in cases 3 and 7 in Fig. 6 is designed to create a green wave for the bus free flow travel time. In this scenario, when buses don't stop at bus stops, the bus delays are lowest, as expected. Hence, TSP benefits are relatively low compared to the other types of offset since most buses would arrive to the intersection during the green. However, when buses dwell at bus stops, this offset is no longer favorable for buses and the TSP strategy can help reduce bus delays. Correspondingly, the changes in arterial cumulative car delay is the relatively high. The above results are also observed for different common cycle lengths.

A final scenario where a bus favorable offset is designed assuming deterministic bus dwell times is tested in cases 4 and 8 shown in

Fig. 6. In this scenario, the offset between intersections is equal to the free flow bus travel time when there is no bus stop and is equal to the free flow bus travel time plus the dwell time when there is a bus stop. In this scenario, the absolute delay of buses is very low when buses dwell at bus stops. Hence, the benefit of TSP is low since most buses pass the intersections during the green period. However, when the cycle length is long, the car delays become very large in this scenario and hinder bus operations, too. Hence, the bus delay increases. Moreover, since buses are stuck in the car queues opportunities to trigger TSP decreases, and hence the benefit of TSP is close to zero.

Overall, these results show that some combinations of offset and cycle length can lead to both high car and bus delays (e.g., case 4 for no bus stops) while others can result in low bus delay and high car delay (e.g., case 4 for mid-block stops), or vice versa (e.g., case 3

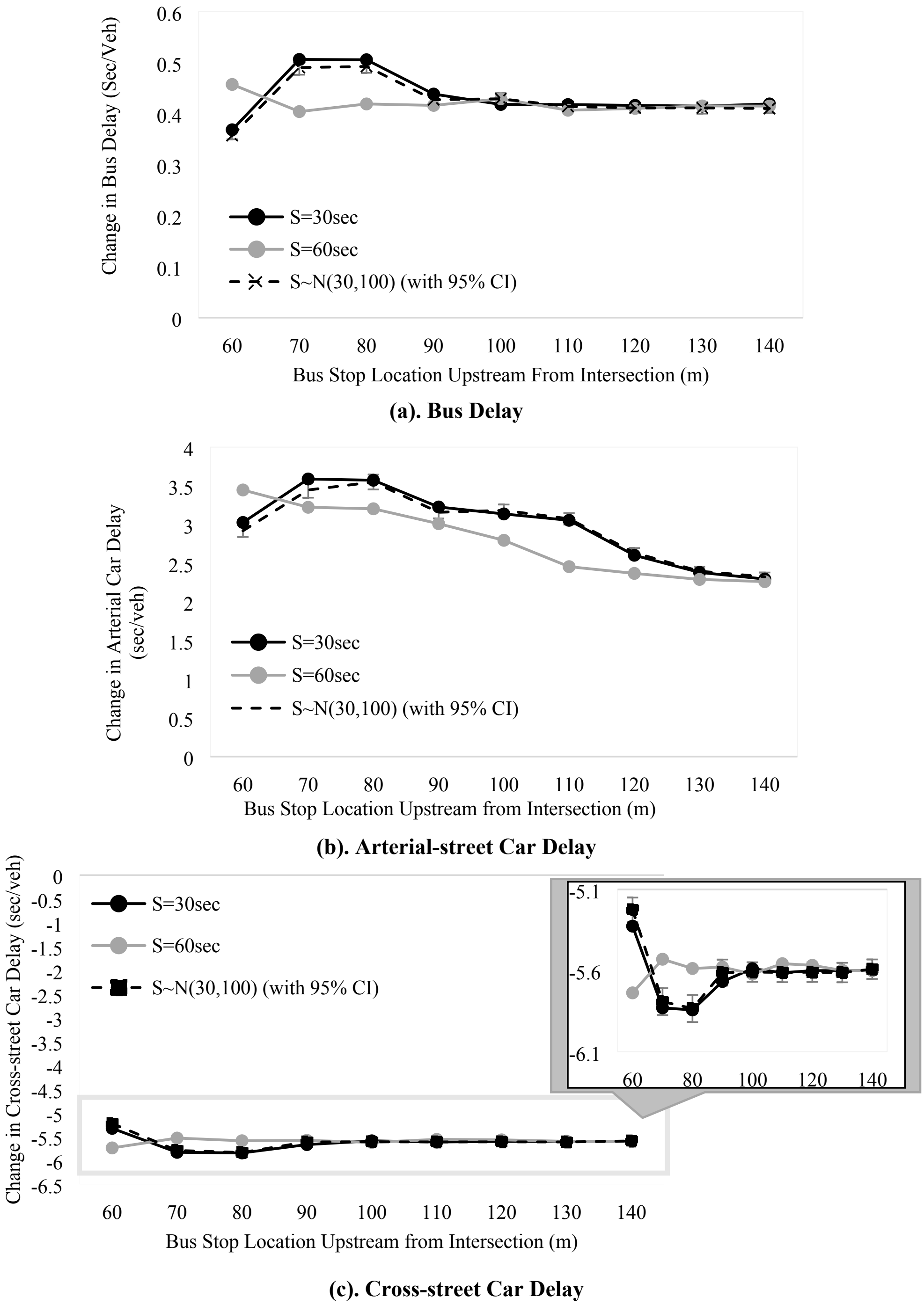

Fig. 7. Sensitivity analysis of bus stop location and dwell duration.

for mid-block stops). Furthermore, TSP can have large benefits on bus delays (and car delays), especially in scenarios designed to favor cars.

4.3. Sensitivity to bus stop location and bus dwell duration

This section considers different combinations of bus stop locations and bus dwell durations to analyze the impacts of TSP implementation. In this experiment, bus stop locations $[60, 140]m$ upstream of the intersection are considered. Note that the distance between intersections is 150 m, hence these can also be considered as $[10, 90]m$ downstream of the intersection. Notice that no bus stops are located after the bus detection zone at $60m$. While this may appear as limiting, 60 m is chosen since it maximizes the accuracy of the bus detection time (for the chosen scenario) and hence helps illustrate the maximum benefit of TSP. Both short ($\mathrm{S} = 30\mathrm{sec}$) and long ($\mathrm{S} = 60\mathrm{sec}$) dwell durations are considered. In general, since passengers do not arrive at bus stops in a deterministic fashion, the bus

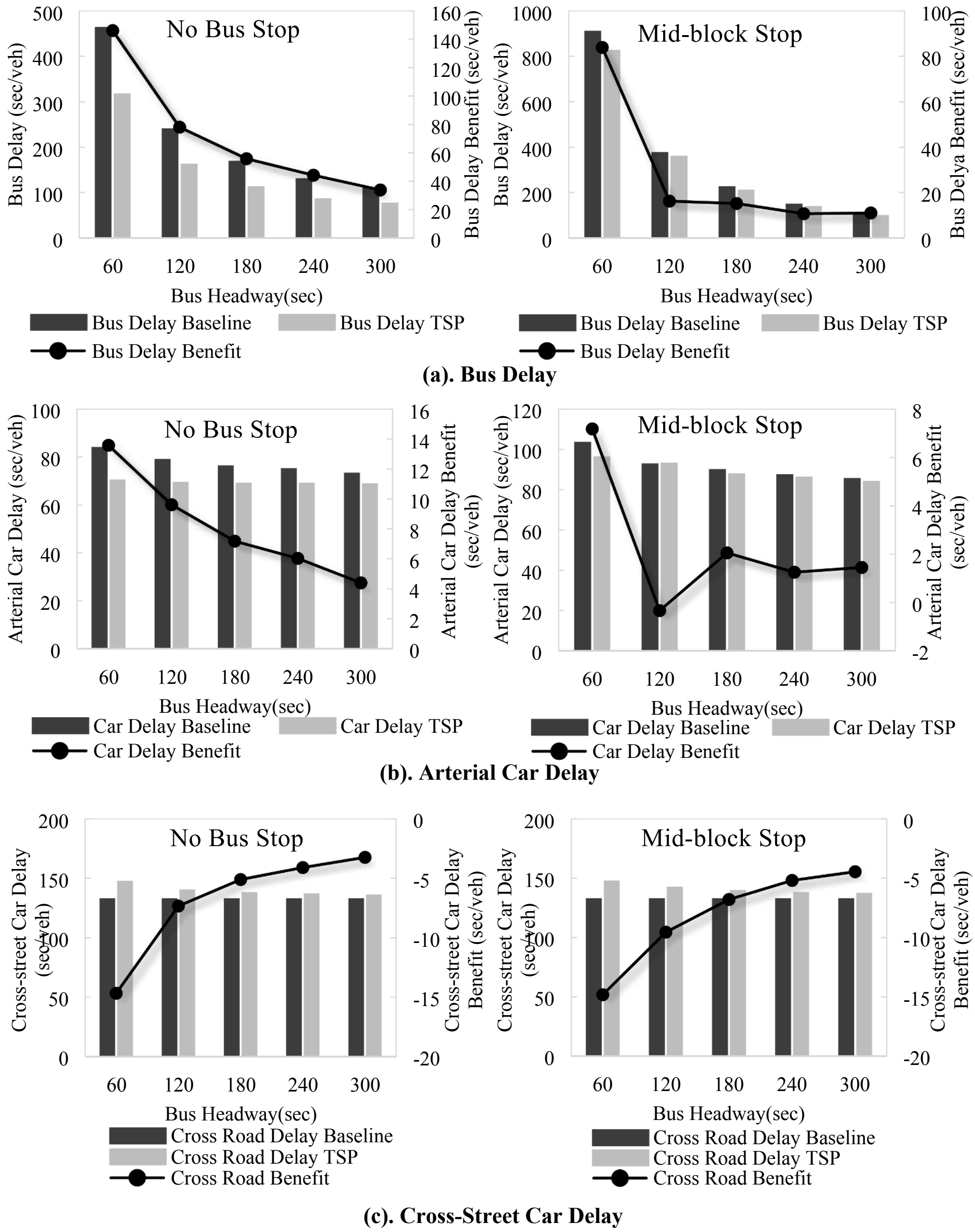

Fig. 8. Sensitivity analysis to bus headway.

dwell times at bus stops could have some randomness. To account for this randomness, tests where the bus dwell time is assumed to follow a normal distribution, $S\ \mathcal{N}(30, 10^2)$, at each bus stop are conducted. In other words, the bus dwell times are drawn from this normal distribution at each bus stop, and the experiment is repeated 30 times using different random seeds to obtain an average result.

The change in bus travel time, arterial street cumulative car delay, cross-street cumulative car delay is shown in Fig. 7a-c. The rectangle markers with error bars in Fig. 7a - c exhibit the means and 95% confidence interval of the experiment results.

For bus stops near the intersection (i.e., $[60, 90]m$ upstream of an intersection), the time when a bus could access the bus stop depends on the length of the queue of cars at a red signal. Hence, TSP can significantly reduce bus delays for some combinations of near-side bus stop placement and dwell duration. For example, in this illustration a bus stop that is placed 60 m upstream of an intersection increases the likelihood of a bus getting stuck in a queue of cars before arriving to the bus stop resulting in it arriving to the bus stop during the green time. Then, a 30 s dwell duration increases the likelihood of this bus departing from the bus stop during the beginning of the red phase, reducing the effectiveness of TSP. Hence, the activation of TSP is sensitive to the combination of near-stop bus stop location and bus dwell duration. Generally, the results show that TSP activation is less sensitive to bus stop location as bus stops are placed further away from the intersection. Hence, the change in bus travel times and cross-street car delay is almost insensitive to mid-block and far-side stops, see Fig. 7a and c.

For arterial street cars, the benefit of TSP to cars decreases as the bus stop moves from near-side to far-side locations. In this case, a dwelling bus at a downstream location causes queue spillback. This bottleneck effect diminishes the benefits of red truncation to cars, since the vehicles following the bus would be delayed at the queue at bus stop locations. Moreover, if buses dwell longer at a bus stop, the benefits diminish even more, see Fig. 7b.

The results of analyzing sensitivity of the results to randomness in dwell time confirms that even though the impacts of TSP on cars and buses are sensitive to the dwell times, the randomness in the dwell times do not change the observed trends. In other words, the mean value of car or bus delay observed when stochastic dwell times are considered are very similar to the results when dwell times are assumed to be deterministic, and the standard error of the average delay is very small.

4.4. Sensitivity to bus headway

The bus headway is defined as the time between two consecutive buses entering the arterial at intersection 1. As the bus headway increases, the number of buses that simultaneously travel within the arterial decreases, hence, the TSP activation frequency decreases. Bus headways of $\boldsymbol{h} = \{60, 120, 180, 240, 300\}sec$ are considered here. Fig. 8a shows the total travel time savings among all buses and per bus, respectively. As can be seen, total bus delay savings decrease as the bus headway increases. However, each individual bus has larger benefit from TSP implementation if there are less buses that travel along the arterial. To explain this phenomenon, as an example consider buses 1 and 2. The TSP activation for bus 1 also benefits cars reducing the residual queues at intersections. This in turn, reduces, the probability of bus 2 being impeded by car queues. Therefore, the marginal benefit per bus decreases as the bus headway decreases. The arterial-street car delay follows a similar trend to buses, as bus headway increases arterial street delays do not increase as much, and vice versa for cross-street cars, see Fig. 8b.

5. Optimal TSP implementation along arterials

In this section, an optimization framework is proposed to determine optimal locations for TSP deployment. Next, a numerical simulation is used to demonstrate some results.

5.1. Optimization of location of TSP implementation

The goal is to find the optimal combinations of intersections that should be equipped with TSP (i.e., have TSP capability). To do so, the objective function is defined as the sum of: 1) weighted bus delay savings, 2) weighted change in arterial street car delay, 3) weighted increase in cross-street car delay, and 4) the cost of implementing TSP, C_T. The decision variable is an array, $\boldsymbol{\tau}$**,** with indicator variables of TSP implementation at each intersection (i.e., $\tau_i = 1$ if TSP is implemented at intersection, i, and $\tau_i = 0$, otherwise). The set of intersections considered for TSP implementation is denoted $\boldsymbol{In}$**,** Then, the mathematical program can be formulated as follows:

$$\text{Min}F = \sum w_1 T_B + w_2 T_B + w_3 T_C^{'} - C_T \tag{18}$$

Subject to:

$$\left[T_B, T_C, T_C^{'}\right] = f(\boldsymbol{\tau}, \boldsymbol{In})$$

$$C_T = g\left(\sum_{i \in I} \tau_i\right)$$

$$\sum_{i \in I} \tau_i \leq \tau_{max}$$

where $T_B, T_C, T_C^{'}$ are the changes in bus travel time, arterial street cumulative car delay, and cross-street car delay, respectively; w_1, w_2,

w_3 are the weights associated with each; C_T is the cost of implementing TSP, which is calculated using is a linear function, $(\cdot)$, since TSP implementation cost is generally proportional to the number of intersections; and τ_{max} is the maximum number of intersections that can be equipped with TSP. The objective function simply considers the weighted sum of the different travel times, where the weights can be assigned based on engineering judgement. Furthermore, the objective function can be modified to account for passenger waiting time at bus stops if average passenger arrival rate at a bus stop information is available.

The problem takes the form of a bi-level optimization program where the lower-level evaluates the different decision variables, i.e., the DP algorithm in Table 1, and the upper level maximizes the weighted summation of benefits (positive value) and costs (negative value) considering different combinations of locations for TSP implementation. This bi-level optimization problem can be solved using a mixed-integer programming algorithm or heuristics such as genetic algorithm. All intersection parameters, bus parameters, fundamental diagram (FD) parameters can be calibrated through field experiments.

5.2. Numerical analysis

Since the weights and cost of TSP implementation are unknown, results of enumerating all possible combinations of TSP locations

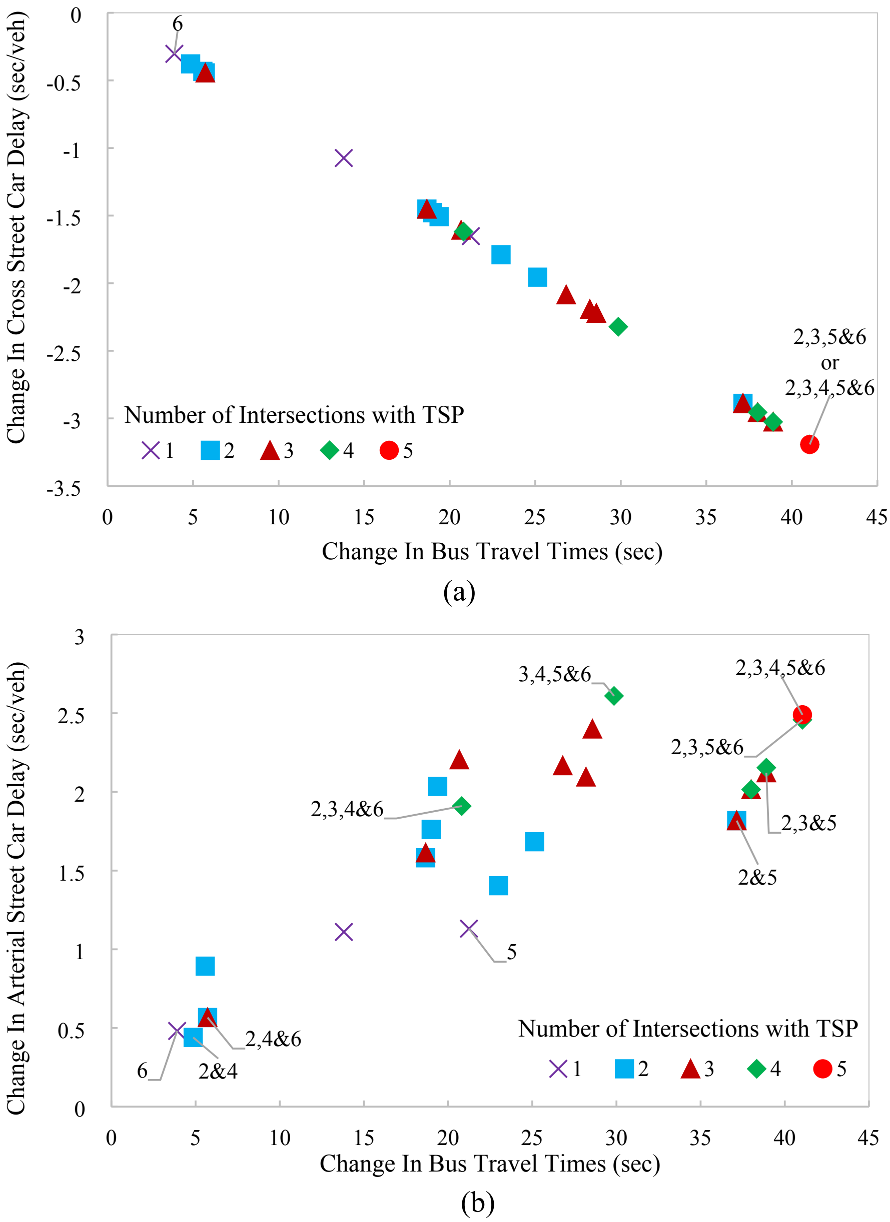

Fig. 9. (a) Change in cross-street car delay versus change in bus travel time; (b) Change in arterial car street delay versus change in bus travel time;

considering the same 7-intersection arterial area shown. It is assumed that there is no TSP implemented at the entrance and exit intersections (i.e., intersections-1 and 7). Only one bus is included in this analysis (i.e., a large headway). Bus arrival times are uniformly distributed across a cycle at intersection 1. The results are shown as an average of all possible bus arrival times.

Fig. 9a shows the change in cross-street car delay, T'_C, versus bus delay savings, T_B, across all intersections. In this figure, the data points corresponding to different number of intersections where TSP is implemented are shown with different color and shape markers. As can be seen, change in cross-street car delay is nearly linearly correlated with bus delay savings. Looking at the results, a 1 s delay saving for buses would impose about 7$\text{veh}\cdot\text{sec}$ in cross-street car delay. This would correspond to the system wide delay (i.e., bus plus car) decreasing if the bus passenger occupancy is more than seven times that of a car. Since this is the case for most bus systems, TSP activation can improve the system as a whole. However, increasing the number of TSP implementations does not always improve bus operations. For example, in this experiment, both implementations at intersections$2, 3, 5, 6$ and $2, 3, 4, 5, 6$ have the same impacts,

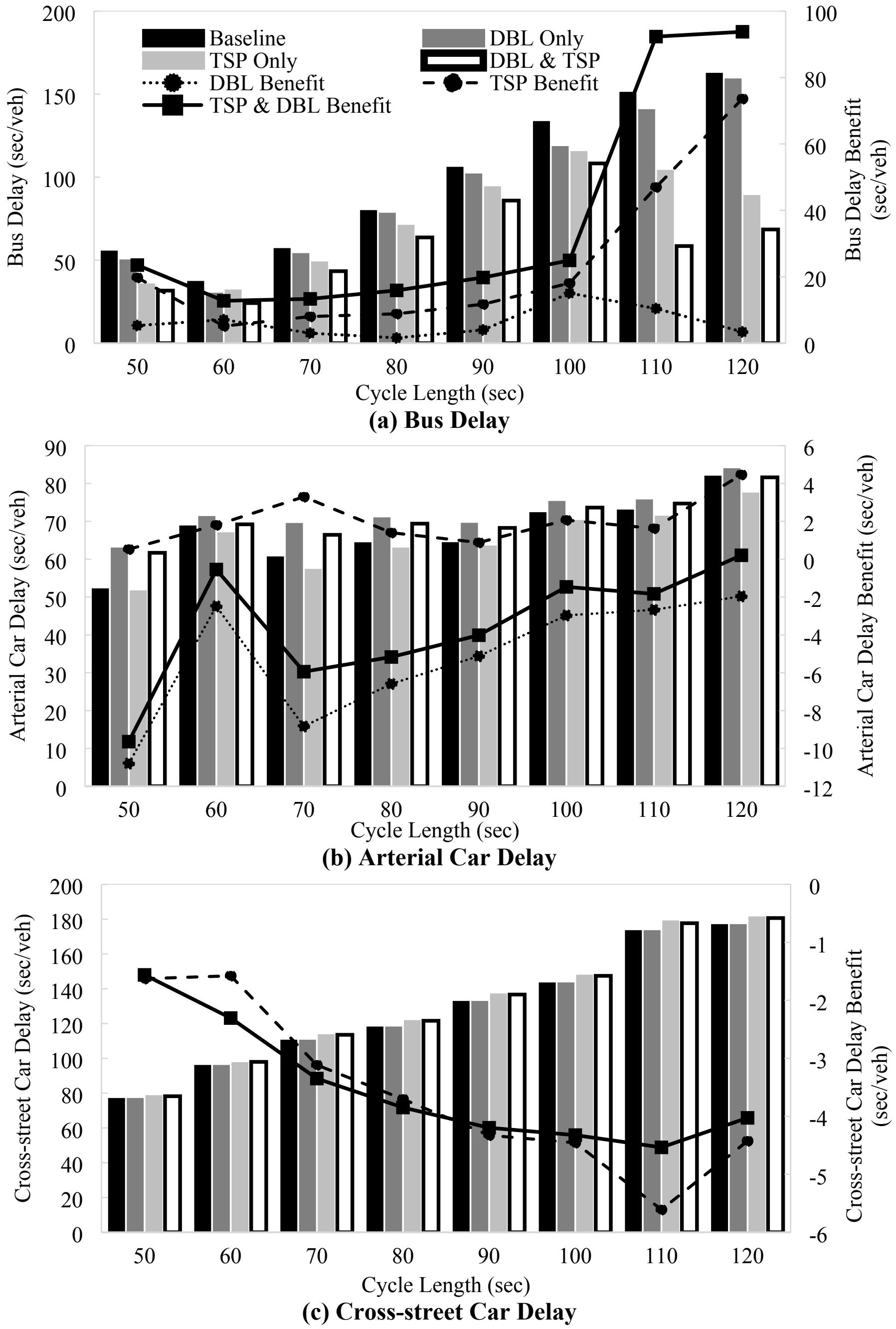

Fig. 10. Sensitivity analysis of combined bus priority strategies.

indicating there is no need to deploy TSP at intersection 4.

Fig. 9b shows the change in arterial-street car delay, T_C, versus bus delay savings, T_B, across all intersections. The labels show the intersection numbers where minimum and maximum bus delay savings are obtained given a certain number of locations equipped with TSP. For example, intersections 2&4 and 2&5 have the minimum and maximum bus delay savings if TSP is simultaneously deployed at two intersections, respectively. This shows that in general if TSP is implemented at more intersections both the bus and car delays can be reduced. However, the results are highly dependent on the location of TSP implementation, since the impacts of TSP vary in a wide range within each group with the same number of TSP implementations. These results also confirm that implementing TSP at some specific intersections can provide the largest benefits, for example, intersection 5 in this experiment since this intersection appears in most of the TSP implementation combinations with maximum bus delay saving.

6. Dedicated bus lanes and TSP

On urban streets combining TSP and DBL could benefit the system, especially for high demand for bus transit. This section further explores the joint impacts of TSP and DBL on both buses and cars using the same 7-intersection arterial, including bus stops. However, now 3 lanes in each direction are assumed to be able to convert a lane to a dedicated bus lane. It is assumed that DBL implementation takes away one travel lane from cars, correspondingly, the variables q_C, k_J, change to be two thirds of their original values. Bus arrival times are uniformly distributed across a cycle at intersection 1 with a headway of 5 mins. The results are shown as an average of all possible bus arrival times. Since bus movement is affected by the signal setting, a series of cycle lengths, $\boldsymbol{C} = \{50, 60, 70, 80, 90, 100, 110, 120\}$ secs are also considered.

Fig. 10a exhibits the change in bus delay for different cycle lengths. When implemented independently the delay savings for DBL only or TSP only are very similar to each other when the cycle length is less than 100 secs. This is because both the DBL and TSP reduces the queuing delay, where DBL can reduce it by providing uninterrupted flow and TSP can reduce it by triggering the red truncation. When the cycle length becomes larger, more buses trigger green extension when TSP is implemented, which can reduce the bus delay significantly. However, the DBL benefits get smaller since the red duration is longer. Additionally, implementation of TSP together with DBL can lead to bus delay savings that are larger than the delay savings of the two individual strategies combined. Hence, the interaction of TSP and DBL can create a synergistic system that significantly benefits the bus. The results found in this paper are consistent with (Truong et al., 2017) that used a numerical model based on the time–space diagram to analyze bus delay when DBL exists. That paper suggests that at the arterial level, with suitable signal offset settings the combined effect of TSP and DBL on bus delay savings can exceed the sum of the individual impacts of the two strategies implemented independently.

Fig. 10b exhibits the corresponding change in arterial-street car delay. As can be seen, the activation of TSP can reduce the delay of cars travelling along the arterial, however, the implementation of DBL significantly increases arterial car delays. Even if TSP is implemented together with DBL, the car delays are still increased significantly. Hence, if bus passenger occupancy is not high, the system-wide benefits of implementing DBL (with or without TSP) might be negative.

Fig. 10c exhibits the change in cross-street car delay. The implementation of DBL imposes no change in signal timing, and hence no change in cross-street car travel time compared to the baseline. On the other hand, the implementation of TSP always increases cross-street car delay compared to the baseline since the green duration of this direction is reduced. However, the presence of a DBL does not significantly change the impact of TSP compared to the baseline since the changes in signal timing remain similar.

7. Conclusions

This study furnishes a bimodal traffic flow modeling approach to evaluate TSP performance within the context of kinematic wave theory assuming mixed traffic on the roadway. A numerical scheme is developed using the Lax-Hopf equation (Aubin et al., 2008), which computes vehicle counts and tracks bus motion in time–space plane with and without TSP. Notice that this model assumes there is no turning movement which is a limitation of this work. Changes in bus travel times and cumulative car delay are both quantified. The algorithm is utilized for sensitivity tests to determine the changes to car and bus delays as a result of TSP implementation, as a function of signal setting, bus stop location and dwell duration, and bus headway. The results suggest that the benefits of TSP largely depend on the signal setting, and bus stop location. However, random dwell times do not change the results. Moreover, the results show that as the bus headway decreases, the marginal benefit of providing TSP also decreases. In practice, not every intersection can be equipped with TSP due to considerable costs of implementation, hence, a bi-level optimization framework is then proposed to determine optimal TSP implementation among intersections. As a result of enumerating all possible combinations of locations for TSP implementation along an arterial, it is found that TSP implementation at certain locations could benefit buses more so than other locations. Finally, the effects of integration of two common bus priority strategies – TSP and DBL is considered. The numerical analysis reveals that while integrated priority can save buses more delay than the two strategies implemented independently, DBL imposes considerable added delay to cars.

The existing work can be furthered by comparing the results to empirical findings, and extending the methodology to a network-level. Additionally, turning movements can be added to the analysis methodology to further improve the realism of the analysis method.

CRediT authorship contribution statement

Kan Wu: Conceptualization, Methodology, Software, Formal analysis, Investigation, Data curation, Writing - original draft.

Muyang Lu: Software, Validation, Investigation, Data curation, Writing - original draft. **S. Ilgin Guler:** Conceptualization, Methodology, Resources, Writing - original draft, Writing - review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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